Engineering Notes

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Theory for the First-Order Gravitational Effects on Ship Forces and Moments in Shallow Water

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1 Introduction

A THEORY for the flow induced by a ship in shallow water has been developed by Newman.[1] This theory, which employs the zero-Froude-number free surface (rigid wall) condition throughout, views the ship as a reflected double body moving laterally between parallel walls. While slenderness considerations are used, the theory goes far beyond simple two-dimensional concepts to account for the fact that the flow goes both under the ship and around the ends.

The problem is analyzed by examining the near field or inner flow as a two-dimensional section in a channel beset by a transverse flow U which is unknown and depends upon the longitudinal coordinate x of the particular section. The local onset flow potential is obtained by recognizing that, for flow in a channel, the constant of integration arising from integrating

$$\partial\phi/\partial y \simeq U \qquad (1)$$

in general depends on x and y in the following way

$$\phi = U[y \pm C(x)] \qquad \epsilon < y < 1 \qquad (2)$$

where the $\pm$ corresponds to $y \gtrless 0$, y being the transverse coordinate and $\epsilon < y < 1$ indicates that this behavior is expected in the region close to the body. In contrast, the potential at large $y \gg 1$ is that of a uniform stream of speed V_y

$$\phi \to V_y Y \qquad y \gg 1$$

Here 1 is of the order of the body length. The flow at the ship, as seen from large distances (of order of ship length), appears to be that about a porous strip in a two-dimensional sense, in the coordinates x and y. By matching the normal and tangential velocity components from the outer and inner representations of the flow at the strip, an integral equation is generated for the unknown local velocity $U(x)$ at the ship. The added mass, lateral force and other quantities of interest are found in terms of U and C using approximate and numerical solutions of this integral equation in Ref. 1.

Data obtained by Fujino[2] and others show that the various coefficients from models operated in shallow water are dependent upon Froude number, some more dependent than others. In any event, the Froude number dependence for hulls in shallow water is much more pronounced than in deep water. Hence, a means for estimating this dependence is needed, and it is to this task that the following analysis is directed.

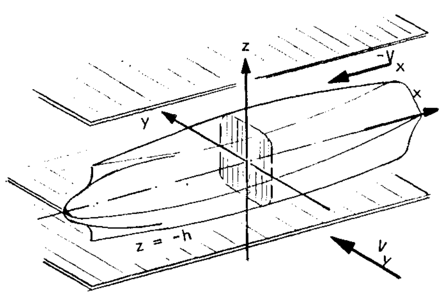

Fig. 1 Definition of coordinates for a ship in shallow water with zero-Froude number image.

Received April 14, 1972.
Index category: Vessel and Control Surface Hydrodynamics; Hydrodynamics.
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2 Shallow Water Theory

We consider the case of a ship in shallow water which is beset by a flow with components $(-V_x, V_y, 0)$ at large distances (order of ship length), see Fig. 1. In the immediate vicinity of any cross section, the ship sees a cross flow of some magnitude $U(x)$ (to retain Newman's notation as much as possible) which is as yet unknown since part of the transverse flow passes under the ship and part around the ends. It has been demonstrated in the literature that such inner flows are to the lowest order, independent of Froude number, the free surface condition being given as $\partial\phi/\partial z = 0$ on $z = 0$ which is the rigid nonporous wall condition. Hence, we may adopt Newman's representation of the potential in the inner field. This is

$$\phi \cong U(x)[y \pm C(x)] \quad \text{for} \quad y \gtrless 0, \epsilon \leq |y| < 1 \qquad (3)$$

where $C(x)$ is a blockage coefficient which must be found from solution of the Neumann problem for each ship section reflected in the free surface placed in a transverse flow between walls representing the sea bottom and its reflection in the free surface. It is assumed that $C(x)$ is known or can be suitably approximated.

The far or outer field representation is constructed to account for that gravitational effect which remains when one suppresses the z-dependence of the flow. We consider the axial flow as the main flow $V_x \gg V_y$ and, upon seeking a first-order velocity potential consistent with slender body concepts which satisfies the Laplace equation, the linearized free surface condition and the bottom condition $\partial\phi/\partial z = 0$, $z = -h$, one obtains the following equation to be satisfied in the outer region (following Tuck[3])

$$(1 - F_h^2)\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = 0; \quad F_h < 1 \qquad (4)$$

where $F_h = V_x/(gh)^{1/2}$, the depth Froude number. This is the familiar linearized form of the compressible flow equation for subsonic Mach numbers, the depth Froude number playing the role of Mach number. We must, therefore, construct the ship potential from a distribution of a fundamental solution of this equation.

When viewed from a distance of the order of a length, the ship in the lateral flow appears to be absorbing fluid. Actually the fluid is passing beneath and around the ship, but we are too myopic to perceive that the lateral flow passes beneath. As far as the far field is concerned, it is as though that part which passes beneath effectively passes through the ship; thus the vessel appears to act like a thin porous barrier extending to the bottom. We must, therefore, construct this barrier from diples or vortices which are simultaneously solutions of Eq. (4).

The potential or Green's function for Eq. (4) which is source-like is

$$\phi_s = m/2\pi k \ln [(x - \xi)^2 + k^2(y - \eta)^2]^{1/2} \quad (5)$$

where $k = (1 - F_h^2)^{1/2}$.

A y-directed shallow water dipole is obtained by differentiating with respect to η, taking the form

$$\phi_d = \sigma k/2\pi\{(y - \eta)/[(x - \xi)^2 + k^2(y - \eta)^2]\} \quad (6)$$

Integration over $\xi = \xi'$ from $-\infty$ to ξ gives the potential of a shallow-water vortex located at $x = \xi$, $\eta = 0$ as

$$\phi_v = (\gamma/2\pi)\cdot\tan^{-1}[(x - \xi)/ky^{(\gamma/2\pi)\tan^{-1}(x-\xi/Ry)}] \quad (7)$$

The ship disturbance potential may then be constructed as a distribution of shallow water vortices of unknown strength density $\gamma(x)$ together with the outer flow potential as

$$\Phi = \frac{1}{2\pi}\int_{-l}^{l} \gamma(\xi)\tan^{-1}\left(\frac{x-\xi}{ky}\right)d\xi + V_y y \quad (8)$$

The velocity components induced by this distribution are

$$u(x, y) = \frac{ky}{2\pi}\int_{-l}^{l}\frac{\gamma(\xi)d\xi}{(x - \xi)^2 + (ky)^2} \quad (9)$$

and

$$v(x,y) = -\frac{k}{2\pi}\int_{-l}^{l}\frac{\gamma(\xi)\cdot(x - \xi)d\xi}{(x - \xi)^2 + (ky)^2} + V_y$$

Along the barrier,

$$u(x + 0) = -(\gamma/2)(x) \qquad |x| \leq l \quad (10)$$

$$u(x - 0) = +(\gamma/2)(x)$$

$$v(x,0) = -\frac{k}{2\pi}\int_{-l}^{l}\frac{\gamma(\xi)d\xi}{x - \xi} + V_y \quad (11)$$

Matching Eq. (10) with the inner approximation at large distances from the inner region ($\epsilon \leq y < 1$) gives

$$\gamma(x) = 2(d/dx)(UC) \quad (12)$$

and

$$v(x) = U(x) = \frac{-k}{2\pi}\int_{-l}^{l}\frac{\gamma(\xi)}{x - \xi}d\xi + V_y \quad (13)$$

Combining Eqs. (12) and (13) gives the following equation for the unknown inner stream velocity $U(x)$

$$U(x) = \frac{k}{\pi}\int_{-l}^{l}\frac{(UC)'}{\xi - x}d\xi + V_y \qquad k = (1 - F_h^2)^{1/2} \quad (14)$$

This equation collapses to that developed by Newman when $k = 1$ or $F_h = 0$ as should be obtained. It is also interesting to note that as $C \to 0$ (large water depth) $U \to V_y$ and the gravitational effect also vanishes. As Newman pointed out, it is the familiar integral equation which arises in calculating the lift on a large aspect ratio wing—in his case for zero Mach number, whereas here it corresponds to the Mach number given by $M = F_h$ when we view U as proportional to the lift coefficient, $C_L/2\pi$, C as $\pi c(x)/4$ (c being the chord) and $V_y = V_x\alpha$, α being the angle of attack.

It is not at all necessary to solve Eq. (13) in order to find how the forces on a ship vary with Froude number. One need only replace $C(x)$ by $C(x)/k$ to reduce the equation to that for zero Froude number. This means, as in the Prandtl-Glauert rule in subsonic flow, that the flow about a ship at Froude number F_h is equivalent to one whose blockage coefficient is increased by the factor $(1 - F_h^2)^{-1/2}$. This result might have been anticipated at the outset from aerodynamic experience. Of course, the rule is limited to moderate F_h and breaks down as $F_h \to 1$.

This enables the direct use of Newman's results to give the formulas for the added mass, added mass moment of inertia, lateral force and moment about the z-axis in the following forms:

Added mass

$$M_{yy} = -\rho\nabla + \frac{2\rho hLB}{(1 - F_h^2)^{1/2}}\int_1^1 \frac{U_0\bar{C}}{V_y}dx \quad (15)$$

Added mass moment of inertia

$$I_{zz} = \frac{-\rho L^3 S_0}{8}\int_1^1 x^2\bar{S}(x)dx + \frac{\rho hL^3B}{(1 - F_h^2)^{1/2}}\int_1^1 \frac{U_0\bar{C}}{L\dot{\theta}}(x)xdx \quad (16)$$

Lateral Force

$$Y = +\frac{4\rho hB}{[(1 - F_h^2)]^{1/2}}\left(\frac{\bar{C}(1)U_0(1)}{V_y}\right)U^2\cdot\beta \quad (17)$$

Moment about z-axis

$$N = \frac{LY}{2} + \frac{\rho}{2}LS_0V_x^2\left\{2C_p + \frac{4}{[(1 - F_h^2)]^{1/2}}\left(\frac{hB}{S_0}\right)\int_1^1\left(\frac{U_0}{V_y}\right)\bar{C}(x)dx\right\}\beta \quad (18)$$

In the above expressions: $\bar{C}$ is the blockage factor divided by ship beam B, S_0 is the maximum cross-sectional area, $\bar{S}$ is the cross-sectional area in fraction of S_0, C_p is the longitudinal prismatic coefficient, and β is the angle of attack $V_y/V_x \ll 1$.

It remains to be seen how well these formulas predict these quantities over a range of Froude numbers and water depths. It is not believed that the terms remaining in the limit $\bar{C} \to 0$ will correctly estimate the infinitely deep results as obtained from model measurements, particularly if the ship model has a skeg at the stern.

References

[1] Newman, J. N., "Lateral Motion of a Slender Body Between Two Parallel Walls," *Journal of Fluid Mechanics*, Vol. 29, 1969, pp. 97–115.

[2] Fujino, M., "Experimental Studies on Ship Maneuverability in Restricted Waters—Part I," *International Shipbuilding Progress*, Vol. 15, No. 168, 1968, pp. 279–301.

[3] Tuck, E. O., "Shallow Water Flows Past Slender Bodies," *Journal of Fluid Mechanics*, Vol. 26, Pt. 1, 1966, pp. 81–95.

Dynamic Similarity Scaling Laws Applied to Cables

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Nomenclature

A = cross-sectional area of cable, L^2

$C_{F,G}$ = hydrodynamic force (F or G) coefficient/length based on cable diameter, 1

D = cable diameter, L

E = modulus of elasticity of cable, F/L^2

F = hydrodynamic force/length normal to cable, F/L

G = hydrodynamic force/length tangent to cable, F/L

M = cable mass in air/length of cable, FT^2/L^2

P = amplitude of motion, L

R = Reynolds number based on cable diameter, 1

T = tension in cable, F

U = velocity of element of cable normal to static cable configuration, L/T

Received September 20, 1971; revision received November 29, 1971.

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